

# Torus or black disk?

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## Abstract

We show that the interaction region of colliding protons looks completely absorptive (black) at the impact parameters up to 0.4 - 0.5 fm at the LHC energy 7 TeV. It is governed by the ratio of the elastic diffraction peak slope to the total cross section. The corresponding parameter is approximately equal to 1 at the LHC. The behavior of this ratio at higher energies will show if this region will evolve to the black disk or to the black torus. Recent fits at 7 TeV can not distinguish between these possibilities within the limits of experimental indefiniteness and of extrapolations in the regions of unmeasured transferred momenta.

The shape of the interaction region of colliding protons changes with increase of their energies. There were arguments that it looks like the completely black disk at asymptotically high energies. Recently it was shown that this conclusion could be misleading. The black region of the size about 0.4 - 0.5 fm is formed at the LHC energy 7 TeV. Its further evolution depends on the energy behavior of the ratio of the elastic diffraction peak slope to the total cross section. The corresponding parameter  $Z = 4\pi B/\sigma_t$  is approximately equal to 1 at the LHC. Both the total cross section of colliding protons  $\sigma_t$  and the slope  $B$  of the differential cross section of elastic scattering increase with energy at high energies.

For the sake of completeness I have to repeat in the beginning some definitions and statements made in my review paper [1] and in my recent paper [2].

The differential cross section of elastic scattering  $d\sigma/dt$  is related to the scattering amplitude  $f(s, t)$  in a following way

$$\frac{d\sigma}{dt} = |f(s, t)|^2. \quad (1)$$

Here  $s = 4E^2$ , where  $E$  is the energy in the center of mass system. The four-momentum transfer squared is

$$-t = 2p^2(1 - \cos \theta) \quad (2)$$

with  $\theta$  denoting the scattering angle in the center of mass system and  $p$  the momentum. The amplitude  $f$  is normalized at  $t = 0$  to the total cross section by the optical theorem such that

$$\text{Im} f(s, 0) = \sigma_t / \sqrt{16\pi}. \quad (3)$$

Note that the dimension of  $f$  is  $\text{GeV}^{-2}$ .

It is known from experiment that protons mostly scatter at rather small angles within the so-called diffraction cone. As a first approximation, it can be described by the exponential shape with the slope  $B$  such that

$$\frac{d\sigma}{dt} \propto e^{-B|t|}. \quad (4)$$

To define the geometry of the collision we must express these characteristics in terms of the transverse distance between the centers of the colliding protons called the impact parameter  $b$ . It is easily done by the Fourier-Bessel transform of the amplitude  $f$  written as

$$i\Gamma(s, b) = \frac{1}{2\sqrt{\pi}} \int_0^\infty d|t| f(s, t) J_0(b\sqrt{|t|}). \quad (5)$$

Using the above formulae, one can write the dimensionless  $\Gamma$  as

$$i\Gamma(s, b) = \frac{\sigma_t}{8\pi} \int_0^\infty d|t| e^{-B|t|/2} (i + \rho(s, t)) J_0(b\sqrt{|t|}). \quad (6)$$

Here  $\rho(s, t) = \text{Re}f(s, t)/\text{Im}f(s, t)$  and the diffraction cone approximation (4) is inserted. Herefrom, one calculates

$$\text{Re}\Gamma(s, b) = \frac{1}{Z}e^{-\frac{b^2}{2B}}, \quad (7)$$

where  $Z = 4\pi B/\sigma_t$  is the variable used in the review paper [1]. This dependence on the impact parameter was used, in particular, in [3].

The elastic scattering amplitude must satisfy the most general principle of unitarity which states that the total probability of outcomes of any particle collision sums to 1 and reads

$$G(s, b) = 2\text{Re}\Gamma(s, b) - |\Gamma(s, b)|^2. \quad (8)$$

The left-hand side called the overlap function describes the impact-parameter profile of inelastic collisions of protons. It satisfies the inequalities  $0 \leq G(s, b) \leq 1$  and determines how absorptive is the interaction region depending on the impact parameter (with  $G = 1$  for full absorption).

It is known from experiment that the ratio of the real part of the elastic scattering amplitude to its imaginary part  $\rho(s, t)$  is very small at  $t = 0$  and, at the beginning, we neglect it and get

$$G(s, b) = \frac{2}{Z}e^{-\frac{b^2}{2B}} - \frac{1}{Z^2}e^{-\frac{b^2}{B}}. \quad (9)$$

For central collisions with  $b = 0$  it gives

$$G(s, b = 0) = \frac{2Z - 1}{Z^2}. \quad (10)$$

Thus, the darkness of the central region is fully determined by the ratio  $Z$ . It becomes completely absorptive only at  $Z = 1$  and diminishes for other values of  $Z$ . The energy evolution of the parameter  $Z$  is shown in the Table 2 of [1]. Here, in the Table, we show the energy evolution of both  $Z$  and  $G(s, 0)$  for  $pp$  and  $p\bar{p}$  scattering.

The function  $G(s, b)$  in Eq. (9) has the maximum at  $b_m^2 = -2B \ln Z$  with full absorption  $G(b_m) = 1$ . Its position depends both on  $B$  and  $Z$ . Note, that, for  $Z > 1$ , one gets  $G(s, b) < 1$  at any physical  $b$  with the largest value reached at  $b = 0$  because the maximum appears at non-physical values of  $b$ . The disk is semi-transparent. At  $Z = 1$ , the maximum is positioned

Table. The energy behavior of  $Z$  and  $G(s, 0)$ .

$\sqrt{s}$ , GeV	2.70	4.11	4.74	7.62	13.8	62.5	546	1800	7000
$Z$	0.64	1.02	1.09	1.34	1.45	1.50	1.20	1.08	1.00
$G(s, 0)$	0.68	1.00	0.993	0.94	0.904	0.89	0.97	0.995	1.00

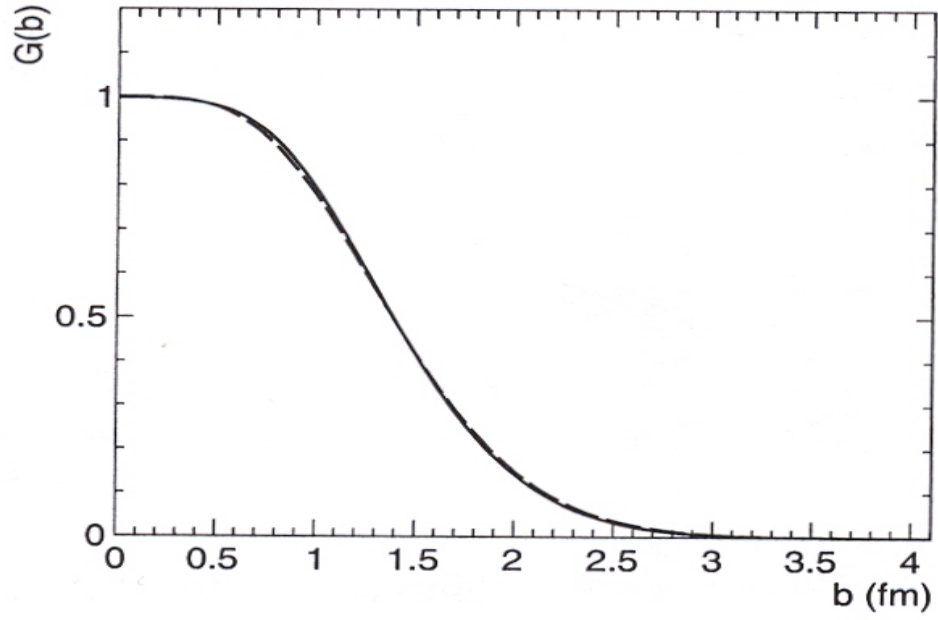


Fig. 1 The impact parameter dependence of the overlap function  $G(b)$  at 7 TeV according to the direct computation from experimental data (solid line) and to the diffraction cone approximation (dashed line). Both curves practically coincide.

exactly at  $b = 0$ , and  $G(s, 0) = 1$ . The disk becomes black in the center. At  $Z < 1$ , the maximum shifts to positive physical impact parameters. The dip is formed at the center. It becomes deeper at smaller  $Z$ . The limiting value  $Z = 0.5$  is considered in more details below.

The maximum absorption in central collisions  $G(s, 0) = 1$  is reached at the critical point  $Z = 1$  which is the case (see the table) at  $\sqrt{s} = 7$  TeV considered first. Moreover, the strongly absorptive core of the interaction region grows in size as we see from expansion of Eq. (9) at small impact parameters:

$$G(s, b) = \frac{1}{Z^2} [2Z - 1 - \frac{b^2}{B}(Z - 1) - \frac{b^4}{4B^2}(2 - Z)]. \quad (11)$$

The second term vanishes at  $Z = 1$ , and  $G(b)$  develops a plateau which extends to quite large values of  $b$  about 0.4 - 0.5 fm. Even larger values of  $b$  are necessary for the third term to play any role at 7 TeV where  $B \approx 20$  GeV<sup>-2</sup>. The structure of the interaction region with a black central core is also supported by direct computation [4] using the experimental data of the TOTEM collaboration [5, 6] about the differential cross section in the region of  $|t| \leq 2.5$  GeV<sup>2</sup>. The results of analytical calculations according to Eq. (9) and the numerical computation practically coincide (see Fig. 1 borrowed from [7]). It was also shown in [7] that this two-component structure is well fitted by the expression with the abrupt (Heaviside-like) change of the exponential. The diffraction cone contributes mostly to  $G(s, b)$ . Therefore, the large- $|t|$  elastic scattering can not serve as an effective trigger of the black core. Inelastic exclusive processes with jets at very high multiplicities can be effectively used for this purpose as shown in [7].

It is usually stated that the equality  $2Z = 8\pi B/\sigma_t = 1$  corresponds to the black disk limit with equal elastic and inelastic cross sections  $\sigma_{el} = \sigma_{in} = 0.5\sigma_t$ . However, one sees from Eq. (10) that  $G(s, b = 0) = 0$  in this case, i.e. the interaction region is completely transparent in the very central collisions. This paradox is resolved if we write the inelastic profile of the interaction region using Eq. (9). At  $Z = 0.5$  it looks like

$$G(s, b) = 4[e^{-\frac{b^2}{2B}} - e^{-\frac{b^2}{B}}]. \quad (12)$$

Recalling that  $B = R^2/4$ , we see that one should rename the black disk as a black torus (or a black ring) with full absorption  $G(s, b_m) = 1$  at the impact parameter  $b_m = R\sqrt{0.5 \ln 2} \approx 0.59R$ , complete transparency at  $b = 0$  and

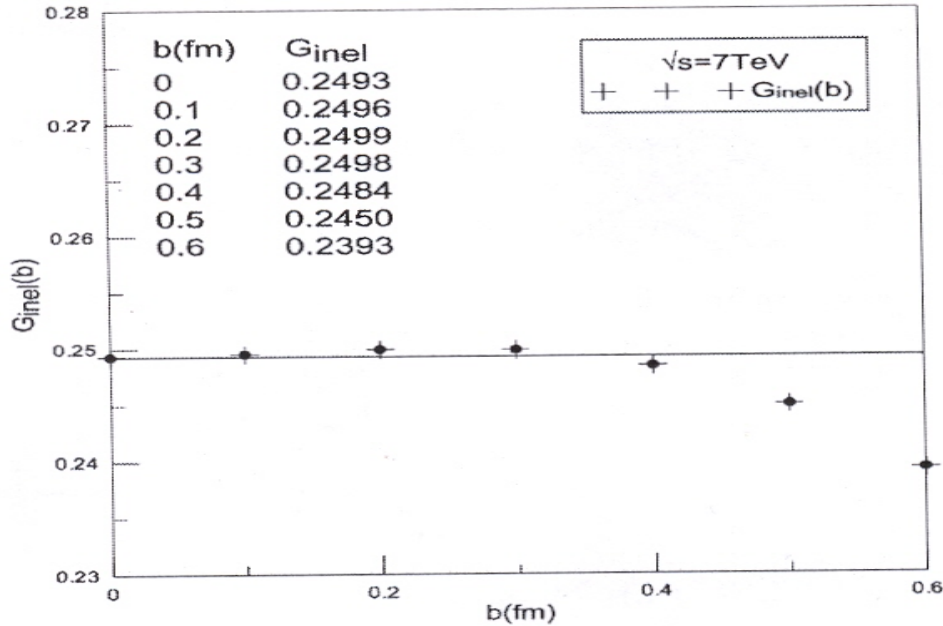


Fig. 2 The impact parameter dependence of the function  $G_{inel}(b) = 0.25G(b)$  at 7 TeV according to the direct computation from experimental data [8].

rather large half-width about  $0.7R$ . Thus, the evolution to values of  $Z$  smaller than 1 at higher energies (if this happens in view of energy tendency of  $Z$  shown in the Table) would imply quite special transition from the two-scale features at the LHC to torus-like configurations of the interaction region. Its implications for inelastic processes are to be guessed and studied.

It is interesting to note that the authors of the paper [8] claim that even at 7 TeV the experimental data show decline from the black disk behavior (see Fig. 2 borrowed from [8]). The transition to the black-torus regime in the described above pattern could be noticed in slight excess at impact parameters 0.1 - 0.3 fm compared to the very center  $b = 0$ . However, it is seen that the excess is so small that it can be explained first by error bars of experimental data. There is no such excess in our results [4, 7] even though the same model was used in both approaches for fits of experimental data. According to our computation the last digits look like 97, 96, 96, 95 for  $b$  from 0 to 0.3 fm so that there is no excess there but the approximate constancy. The minor difference in conclusions attributed to the last digits in

the numbers shown in Fig. 2 for  $G_{inel}(b)$  can be ascribed to different procedures adopted in these papers for extrapolations to the ranges of transferred momenta where there are no experimental data yet.

Thus it seems too early to make any (even preliminary) statements. However, the comparison of the results of [4, 7] and [8] shows that we are in the critical regime of elastic scattering at 7 TeV as stressed in [2] and should pay special attention to evolution of the parameter  $Z$  at higher energy of 13 TeV which will become available soon.

To conclude, we have shown that the shape of the interaction region of two protons colliding at high energies evolves with energy and becomes critical at 7 TeV. The absorption at the center of the interaction region of protons is determined by a single energy-dependent parameter  $Z$ . The region of full absorption extends to quite large impact parameters if  $Z$  tends to 1 that happens at  $\sqrt{s} = 7$  TeV. Its difference from 1 at this energy can not be determined with high enough precision up to now to decide definitely if the tendency to the new regime has been observed already. Therefore, the energy behavior of  $Z$  at higher energies is especially important in view of possible evolution of the geometry of the interaction region.

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